GCSE Maths – Algebra

Rearranging Formulae

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of questions regarding rearranging formulae. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

The temperature conversion formula between Fahrenheit, F, and Celsius, C, is given by $F = \left(C \times \frac{9}{5}\right) + 32$. What is 40 degrees Celsius in Fahrenheit?

Step 1: Write down all the given values and then substitute these values into the given formula.

Given:
$C = 40\ degrees\ celsius$

We do not need to rearrange the formula since the subject of the formula is F which is what we want to find. Substitute $C = 40$ into the formula:

$$F = \left(40 \times \frac{9}{5}\right) + 32$$

Step 2: Calculate the value asked in the question. This should be the subject of the formula.

$$F = \left(40 \times \frac{9}{5}\right) + 32 = 104$$

*Therefore, 40 degrees Celsius is **104 degrees Fahrenheit**.*

Guided Example

Consider the equation $a = \frac{v-u}{t}$.
Calculate acceleration when $v = 105$, $u = 25$ and $t = 5$.

Step 1: Write down all the given values and then substitute these values into the given formula

$v = 105$
$u = 25$
$t = 5$

$a = \frac{v-u}{t} = \frac{105-25}{5}$

Step 2: Calculate the value asked in the question. This should be the subject of the formula.

$a = \frac{105-25}{5} = \frac{80}{5} = 16$

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

1. Use substitution in the following questions to work out the value of the required variable.

a) Given $x = 54$ and $y = 10$, calculate z when $z = \frac{xy}{3} + 3y$.

$x = 54 \quad y = 10$

$z = \frac{xy}{3} + 3y = \frac{(54)(10)}{3} + 3(10)$

$= \frac{540}{3} + 30 = 210$

b) Consider the equation $a = 4(x + 8) + b$. Calculate a when $x = 10$ and $b = 33$.

$x = 10 \quad b = 33$

$a = 4(x+8) + b = 4(10+8) + 33$

$= 4(18) + 33$

$= 105$

c) A sailor uses the following equation to calculate when their ship will reach its destination:

$$t = 3(x + b^2) - 45$$

Suppose t and x are time measured in hours and b is the number of the people on the ship. Find t when $x = 100$ minutes and the number of people on the ship is 6.

$x = 100 \text{ min} = \frac{100}{60} = \frac{5}{3}$ hours $\quad b = 6$

$t = 3(x + b^2) - 45$

$= 3(\frac{5}{3} + 6^2) - 45$

$= 3(\frac{5}{3} + 36) - 45$

$= 3(\frac{113}{3}) - 45$

$= 113 - 45$

$= 68$ hours

Section B

Worked Example

A trapezium has an area of 60 m^2. The sum of its parallel sides is 24 m. Find the height of the trapezium. The formula for the area of a trapezium is $A = \frac{(a+b)h}{2}$.

Step 1: Rearrange the formula to make the required variable the subject.

$$A = \frac{(a+b)h}{2}$$

We are finding h so we want to make h the subject of the equation:

Multiply both sides of the equation by 2:

$$2A = (a+b)h$$

Divide both sides of the equation by $(a+b)$:

$$h = \frac{2A}{a+b}$$

Step 2: Write down all the given values and substitute these values into the formula to calculate the required value.

Given values: $A = 60,\ a+b = 24$

$$h = \frac{2A}{a+b} = \frac{2(60)}{24} = 5 \text{ m}$$

Guided Example

Suppose $x = y^2 + 2ab$. Find b when $x = 25, y = 3$ and $a = 4$.

Step 1: Rearrange the formula to make the required variable the subject.

Finding b:

$-y^2$ | $x = y^2 + 2ab$

$x - y^2 = 2ab$

$\div 2a$ | $\frac{x-y^2}{2a} = b$

Step 2: Write down all the given values and substitute these values into the formula to calculate the required value.

$x = 25 \quad y = 3 \quad a = 4$

$$b = \frac{x-y^2}{2a} = \frac{25-3^2}{2(4)} = \frac{16}{8} = 2$$

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

2. Rearrange the following equations to make x the subject:

a) $y = 403x + 5$

-5 | $y - 5 = 403x$

$\div 403$ | $\frac{y-5}{403} = x$

$x = \frac{y-5}{403}$

b) $Q = 35(x + 5)$

$\div 35$ | $\frac{Q}{35} = x + 5$

-5 | $\frac{Q}{35} - 5 = x$

$x = \frac{Q}{35} - 5$

c) $H = \frac{30}{x} + 34y$

$-34y$ | $H - 34y = \frac{30}{x}$

$\times x$ | $x(H - 34y) = 30$

$\div (H - 34y)$ | $x = \frac{30}{H-34y}$

$x = \frac{30}{H-34y}$

3. Consider the equation $4(x + y) = 3x + 3$.

a) Rearrange this equation to make y the subject.

$\div 4$ | $x + y = \frac{3x+3}{4}$

$-x$ | $y = \frac{3x+3}{4} - x$

b) Find the value of y when $x = 2$

$x = 2$

$y = \frac{3(2)+3}{4} - 2$

$= \frac{6+3}{4} - 2 = \frac{1}{4}$

Section C

Worked Example

Make m the subject of the equation $n = \frac{m-4}{m-13}$.
Find the value of m, when $n = 9$.

Step 1: Try to eliminate the fraction.

We are making m the subject.

Multiply both sides by $(m - 13)$ and expand any brackets:

$$n = \frac{m-4}{m-13}$$

$$n(m-13) = m-4$$

$$nm - 13n = m - 4$$

Step 2: Bring all the terms with the subject onto one side and then factorise the subject out of the terms.

Bring both the terms containing m onto the left-hand side of the equation by subtracting m from both sides of the equation. Add $13n$ to both sides of the equation to move the terms without m to the right-hand side of the equation.

$$nm - 13n = m - 4$$

$$nm - 13n - m = -4$$

$$nm - m = 13n - 4$$

$$m(n-1) = 13n - 4$$

Step 3: Divide by the bracket which the subject has been factored out of to obtain an expression with only the subject on one side of the equation.

$$m(n-1) = 13n - 4$$

$$m = \frac{13n-4}{n-1}$$

Step 4: Write down all the given values and substitute these values into the formula.

Given values: $n = 9$

$$m = \frac{13n-4}{n-1} = \frac{13(9)-4}{9-1} = \frac{113}{8}$$

So, when $n = 9$, $m = \frac{113}{8}$.

Worked Example

Make u the subject of the equation $\frac{13}{a} = \frac{1}{b-1} - \frac{1}{u}$.

Step 1: Aim to eliminate all the fractions, by multiplying the expression by the denominators.

Multiply the equation by a:

$$13 = \frac{a}{b-1} - \frac{a}{u}$$

Multiply the equation by $(b-1)$:

$$13(b-1) = a - \frac{a(b-1)}{u}$$

Multiply by u:

$$13u(b-1) = au - a(b-1)$$

Step 2: Bring all the terms with the subject onto one side and then factorise the subject out of the terms.

$$13u(b-1) = au - a(b-1)$$

$$13u(b-1) - au = -a(b-1)$$

$$u(13(b-1) - a) = -a(b-1)$$

Step 3: Divide by the bracket which the subject has been factored out of to obtain an expression with only the subject on one side of the equation.

$$u(13(b-1) - a) = -a(b-1)$$

$$u = \frac{-a(b-1)}{13(b-1)-a}$$

$$\boldsymbol{u = \frac{-ab-a}{13b-13-a}}$$

Guided Example

Make s the subject of the equation $j = \frac{2s-1}{s-2}$.

Step 1: Aim to eliminate the fraction.

$$j(s-2) = 2s-1$$
$$\Rightarrow js - 2j = 2s - 1$$

Step 2: Bring all the terms with the subject onto one side and then factorise the subject out of the terms.

$$js - 2s = -1 + 2j$$
$$s(j-2) = -1+2j$$

Step 3: Divide by the bracket which the subject has been factored out of to obtain an expression with only the subject on one side of the equation.

$$s = \frac{-1+2j}{j-2}$$

Guided Example

Make q the subject of the equation $\frac{1}{d} = \frac{1}{f} + \frac{3}{3q}$.

$$\frac{1}{d} = \frac{1}{f} + \frac{1}{q}$$

Step 1: Aim to eliminate the fraction.

$$\frac{1}{f} + \frac{1}{q} = \frac{q+f}{fq} \quad \text{[Cross Multiply]}$$

$$\frac{1}{d} = \frac{q+f}{fq}$$
$$fq = d(q+f)$$

Step 2: Bring all the terms with the subject onto one side and then factorise the subject out of the terms.

$$fq = dq + df$$
$$-dq \quad fq - dq = df$$
$$q(f-d) = df$$

Step 3: Divide by the bracket which the subject has been factored out of to obtain an expression with only the subject on one side of the equation.

$$q = \frac{df}{f-d}$$

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

4. Rearrange the following equations to make x the subject.

a) $y = \frac{9x-4}{13-x}$

$-13y \quad -9x$

$y(13-x) = 9x-4$

$13y - xy = 9x - 4$

$-9x - xy = -4 - 13y$

$x(-9-y) = -4-13y$

$x = \frac{-4-13y}{-9-y}$

b) $3 - y = \frac{1-x}{3-9x}$

$+x \quad -1$

$(3-y)(3-9x) = 1-x$

$3(3-9x) - y(3-9x) = 1-x$

$9 - 27x - 3y + 9xy = 1-x$

$8 - 26x - 3y + 9xy = 0$

$-26x + 9xy = 3y - 8$

$x(-26+9y) = 3y-8$

$x = \frac{3y-8}{-26+9y}$

c) $y - 3 = \frac{13gx-5}{8-x}$

$(y-3)(8-x) = 13gx - 5$

$y(8-x) - 3(8-x) = 13gx - 5$

$8y - xy - 24 + 3x = 13gx - 5$

all x terms stay on one side

$-xy + 3x - 13gx = -8y + 24 - 5$

$x(-y + 3 - 13g) = -8y + 19$

$x = \frac{-8y+19}{-y+3-13g}$

d) $\frac{j}{t} = \frac{3x-2+4x}{4-2x}$

$j(4-2x) = t(3x-2+4x)$

$4j - 2jx = t(7x-2)$

$4j - 2jx = 7tx - 2t$

$-2jx - 7tx = -2t - 4j$

$x(-2j-7t) = -2t - 4j$

$x = \frac{-2t-4j}{-2j-7t}$

LHS: Left hand side → RHS: Right hand side

e) $\frac{1}{3y} = \frac{4}{4x-3} - \frac{3}{3c}$

RHS: → $\frac{4}{4x-3} - \frac{3}{3c} = \frac{4(3c) - 3(4x-3)}{3c(4x-3)}$

Subbing into original equation: $\frac{1}{3y} = \frac{12c - 12x + 9}{3c(4x-3)}$

Cross multiply:

$3c(4x-3) = 3y(12c-12x+9)$

$12xc - 9c = 36yc - 36xy + 27y$

$12xc + 36xy = 36yc + 27y + 9c$

$x(12c + 36y) = 36yc + 27y + 9c$

$$x = \frac{36yc + 27y + 9c}{12c + 36y}$$